Simulation of Electromagnetic Radiation and Scattering Using Hybrid Higher Order FETD-FDTD Method

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Abstract The stable hybrid Finite Element Time Domain - Finite Difference Time Domain (FETD-FDTD) method is extended by incorporating higher order hierarchical basis functions in the finite element region. The use of unstructured tetrahedral elements in the modeling of antenna structure enables the application of the hybrid method to accurately model geometrically complex radiators. Pyramidal elements are used in the transition from unstructured tetrahedral elements to structured hexahedral elements of the FDTD grid. The finite element formulation incorporates the excitation of antennas using coaxial line or stripline feed with Transverse Electromagnetic Mode (TEM). Traditional FDTD method with anisotropic Perfectly Matched Layer (PML) is used to simulate unbounded media. The technique is extended for scattering problems, enabling the modeling and simulation of reception by antennas. Application of this method in the modeling of typical problems, enabling the modeling and simulation of reception by unbounded media. The technique is extended for scattering anisotropic Perfectly Matched Layer (PML) is used to simulate Electromagnetic Mode (TEM). Traditional FDTD method with using coaxial line or stripline feed with Transverse Electromagnetic Mode (TEM).

I. INTRODUCTION

The hybrid FETD-FDTD method [3] retains the efficiency of FDTD method in modeling simple geometries while using unstructured FETD method to eliminate stair-casing errors in modeling complex geometries. In this paper, we present the development of hybrid FETD-FDTD method with higher order basis functions in the FETD region [4] and its application for problems of both radiation and scattering. The requirement on mesh generation and a simple strategy that we use are briefly discussed. Modeling of antenna feed structure which excite transverse electromagnetic(TEM) mode and extraction of input reflection coefficient is presented. Finally, application of the method in modeling of radiation from antennas and scattering from arbitrary geometries is presented.

A. Hybrid FETD-FDTD Technique

The first step in the hybrid mesh generation involves basic unstructured tetrahedral mesh generation of the antenna/scattering structure. The outer boundary of the tetrahedral mesh must have a surface triangulation consistent with the FDTD cell size, Δh. A layer of hexahedral elements with edge length Δh is then added around the tetrahedral mesh. In the interface between tetrahedral and hexahedral elements, the non-conforming (tetrahedral) edges must be one of the diagonals of the rectangular faces of the hexahedral elements on the interface. The number of nodes on the interface from the tetrahedral mesh and the hexahedral mesh are the same. The number of edges on the tetrahedral interface is greater than the number of edges on the hexahedral interface by the number of rectangular faces on the interface. Once such a mesh is generated, hexahedral elements with a face on the interface is split into two tetrahedra and five pyramidal elements leading to the final hybrid mesh. Edge elements have been used extensively on tetrahedral and hexahedral elements. For the 3D hybrid FETD-FDTD method, similar basis functions need to be defined over pyramidal elements. Edge vector basis functions on pyramidal elements are available in [5]. Each basis function is associated with a particular edge of the pyramidal element, similar to edge vector basis functions on tetrahedral elements. Thus, each pyramidal element has eight degrees of freedom, corresponding to the number of edges.

B. Higher Order Hierarchical Basis Functions

Higher order vector basis functions provide an advantage of improved resolution of the field solution along with the benefit of having coarser elements in the finite element mesh. Hierarchical higher order vector basis functions [6] provide an added advantage of locally resolving the fields by using basis functions of different orders within a computational domain. The edge element basis functions are the lowest order tangential vector finite elements incomplete to the order 1. They are also referred to as having a mixed-order of 0.5 [7]. Edge elements span the space $H^1(\text{curl};\Omega)$ where superscript 0 refers to the highest degree of polynomial of the curl of the basis function. The curl of the edge element on a tetrahedron is a constant vector function. The next set of higher-order basis functions with the corresponding curl being piecewise-linear vector functions, excluding the gradient fields [8], span the space $H^1(\text{curl};\Omega)$. The next set of higher-order basis functions with the corresponding curl being piecewise-linear vector functions, excluding the gradient fields [8], span the space $H^1(\text{curl};\Omega)$.

TABLE I

<table>
<thead>
<tr>
<th>Order</th>
<th>Topology</th>
<th>Basis Functions</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Edge {i, j}</td>
<td>$\xi_i \nabla \xi_j - \xi_j \nabla \xi_i$</td>
<td>6</td>
</tr>
<tr>
<td>1.5</td>
<td>Edge {i, j}</td>
<td>$\xi_i \nabla \xi_j + \xi_j \nabla \xi_i$</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Face {i, j, k}</td>
<td>$4\xi_i (\xi_j \nabla \xi_k - \xi_k \nabla \xi_j)$</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Face {i, j, k}</td>
<td>$4\xi_j (\xi_i \nabla \xi_k - \xi_k \nabla \xi_i)$</td>
<td>4</td>
</tr>
</tbody>
</table>


tangential vector basis functions, their associated topology and dimensions for a tetrahedral element.
With the gradient basis functions eliminated, these basis functions are incomplete to the order 2 [6] and also referred to as having a mixed-order of 1.5 [7]. For a tetrahedral element these basis functions are given as in Table I where \( \tilde{\varphi}_i \) is the scalar linear Lagrange interpolation polynomial associated with node \( i \). It is to be noted that the basis functions in Table I are not unique and is possible and valid to employ them in other forms as proposed in [6,7]. The choice of basis functions in Table I were used in [9]. With the higher order basis functions, each edge of the tetrahedron has two basis functions associated with it, one of them being the lower order edge element basis function. Similarly each face of the tetrahedron has two basis functions associated with it. Each basis function has tangential component only along its associated topology listed in Table I and has zero tangential component along other non-associated edges and faces. The evaluation and assembly of mass and stiffness matrices for arbitrary tetrahedral elements can be performed analytically and efficiently by constructing universal matrices following the procedure discussed in [6].

In the hybrid FETD-FDTD method, it is possible to use hierarchical higher order basis functions throughout the FETD mesh, on all hexahedral, pyramidal and tetrahedral elements. However, the FDTD region surrounding the unstructured FETD region has lower order edge elements, demanding a typical FDTD grid size of \( \lambda_{nud}/20 \) where \( \lambda_{nud} \) is the smallest wavelength over the frequencies of interest. Due to the conformal nature of the FDTD grid and FETD mesh, the hexahedral and pyramidal elements adjacent to the FDTD region have the same edge lengths as the FDTD grid. Often, for lower order, edge element basis functions the grids size should be around \( \lambda_{nud}/20 \), and it is inefficient and unnecessary to have higher order basis functions on such elements. Due to this fact, the use of higher order basis functions is restricted to tetrahedral elements alone. The hierarchical nature of the basis functions enables the use of higher order bases on the tetrahedral elements while lower order edge elements are used in the pyramidal and hexahedral elements. It is to be noted that the tetrahedral elements adjacent to the pyramidal elements share a common face and three common edges. No degree of freedom is assigned to the faces shared by pyramidal and tetrahedral elements. Similarly no degree of freedom corresponding to the higher order basis functions associated is assigned to the edges shared by pyramidal and tetrahedral elements. This way, vector basis functions of different orders can be used within one finite element mesh. In Fig. 1, the basis functions associated with a tetrahedral element sharing a face with a pyramidal element and an edge with a hexahedral element is shown.

C. TEM Port Excitation

The FETD formulation includes excitation of ports using TEM mode. An absorbing boundary condition is applied for TEM mode and higher order modes (which are evanescent) are ignored. In the time-harmonic case, the total electric fields inside a transmission line exciting a TEM wave in the +z direction is given by

\[
\vec{E} = E_0 \vec{e}_{TE} e^{-j\beta z} + \Gamma E_0 \vec{e}_{TE} e^{j\beta z}
\]

(1)

where \( \vec{e}_{TE} \) is the modal field distribution, \( E_0 \) is the incident field amplitude, \( \Gamma \) is the reflection coefficient if TEM mode and \( \beta \) is the propagation constant of the TEM mode. For dielectric filled coaxial lines and striplines, \( \beta = \sqrt{\varepsilon_r \omega / c} \) where \( \varepsilon_r \) is the dielectric constant of the medium. From (1), the time-dependent boundary condition for the electric field at the port can be derived as

\[
\vec{\varepsilon} \times \nabla \times \vec{E} = \frac{\sqrt{\varepsilon_r}}{c} \vec{\varepsilon} \times \frac{\partial}{\partial t} \vec{E}
\]

(2)

The Galerkin testing of the time-dependent vector Helmholtz equation with (2) as the boundary condition on ports, followed by application of edge element basis functions defined over tetrahedral, pyramidal and hexahedral elements to expand the electric field as

\[
\vec{\varepsilon}_E(t) = \sum_{i=1}^{N} \xi_i(t) \vec{W}_i
\]

leads to the following system of ordinary differential equation viz.,

\[
S_\varepsilon + \frac{1}{c^2} \frac{d}{dt} e + \frac{1}{c^2} \frac{d^2}{dt^2} e - \frac{\varepsilon_r}{c^2} \frac{d}{dt} E_0(t) = 0
\]

(3)

where

\[
S_{\varepsilon}(i,j) = \int_\Omega \nabla \times \vec{W}_i \cdot \nabla \times \vec{W}_j \ d\Omega,
\]

\[
T_{\varepsilon}(i,j) = \int_\Gamma \vec{W}_i \cdot \nabla \times \vec{W}_j \ d\Gamma
\]

\[
B_{\varepsilon}(i,j) = \int_\Omega (\nabla \times \vec{W}_i) \cdot \nabla \times (\nabla \times \vec{W}_j) \ d\Omega,
\]

\[
T_{\varepsilon}(i) = \int_\Gamma (\nabla \times \vec{W}_i) \cdot (\nabla \times \vec{W}_j) \ d\Gamma
\]

and \( E_0(t) \) is the excitation waveform with significant spectral contents in the frequency band of interest. The modal distribution \( \vec{\varepsilon}_E \) is obtained by a 2D finite element eigenvalue solver. For the 2D problem, the triangulation of the port surface in the tetrahedral mesh is employed as the finite element mesh. The temporal discretization of (3) is performed using the \( \Theta \)-method as discussed in [1]. Using the orthogonality property of the modes, the reflection coefficient in (1) is obtained as

\[
\Gamma(\omega) = \frac{1}{F(E_0(t))} \int_{\Gamma_P} \vec{E}_0(t) \cdot \vec{\varepsilon}_E(t) ds - 1
\]

(4)

where \( F(u(t)) \) is the Fourier transform of \( u(t) \).

D. Computation of Time-Domain Scattering

To extend the hybrid method to simulate transient scattering from antennas and structures the formulation should incorporate excitation by a plane wave incident in an arbitrary direction. There are two possible ways to simulate
the excitation by a planewave. One method is to incorporate the traditional, well established total-field/scattered-field (TF/SF) boundary condition in the FDTD grid on which an Huygen’s planewave source is applied. Alternatively, the second method is to employ the Total/Scattered Field Decomposition (TSFD) technique [11] in the FETD grid. It is to be noted that the volumetric based TSFD technique has better accuracy than the TF/SF technique used in FDTD grid but is computationally more expensive due to the need to project the incident field onto the unstructured total-field grid for each time step. TF/SF boundary condition in the FDTD grid on the other hand has a marginally higher error but is efficient and straightforward to implement and hence is employed in this study. In the scattered field region, by either applying time-domain or frequency-domain near-field to far-field (NFFF) transformation, the scattered far-field solution and hence the radar cross section of the structure can be computed.

II. NUMERICAL EXAMPLES

A. Balanced Anti-podal Vivaldi Antenna

The first example is the modeling of a wideband anti-podal Vivaldi antenna. The simulated problem was set as the 2000 CAD benchmark problem by Microwave Engineering Europe [10]. The requirement was to compute the reflection coefficient of the stripline fed antenna structure shown in Fig. 2 in the frequency band 0.5-10GHz. From the geometry it is seen that both the microstrip line and ground plane gradually flare out. The antenna is a tri-plate structure with the substrate being 40mm×90mm with a dielectric constant, ε_r=2.32. The combined thickness of the two sandwiched substrates is 3.15mm and the antenna is fed by a stripline port of dimension 12mm×3.15mm. The details of the antenna geometry with elliptic output flares can be found in [9]. Δh is set as 2mm. Also shown in Fig. 2 is the triangulation of PEC surface of the antenna geometry in the final finite element mesh. Solution using both edge element and higher order basis functions are obtained using the same hybrid mesh. The number of unknowns in the FETD region is 119,898 in the case of the edge elements, and is 657,546 in the case of higher order basis functions for tetrahedral elements. For the higher order solution, complete Cholesky factorization with re-ordering takes 48min and the time taken for time-stepping 5000 time steps is 4hrs on a Dual CPU 64bit AMD Opteron machine. The reflection coefficient at the port obtained using both edge element and higher order basis functions is compared with the results obtained using Ansoft HFSS® in Fig.3. The higher order solution has much better agreement with results from HFSS than the lower order edge element solution. The time domain modal electric field amplitude of the reflected signal at the stripline port obtained with edge element and higher order basis functions is compared in Fig. 4. Though a similar trend is observed in the two waveforms the lower order solution in general lags behind the more accurate higher order solution indicating the dependence of the numerical dispersion characteristics on the order of basis functions. It is to be noted that the HFSS solution in the benchmark exercise was obtained using adaptive mesh refinement while the FETD-FDTD solution with higher order basis functions proposed in this paper is obtained by a crude increase in the order of basis functions in the FETD region.
B. Scattering by a PEC Cube

The second example illustrates the use of the technique for the modeling and computation of transient electromagnetic scattering. The structure is a simple PEC cube of dimension 1cm×1cm×1cm. Δh is set as 1mm. The finite element region has 59,466 unknowns for the lower order edge element case and 219,300 unknowns for the case of higher order basis functions. The bistatic RCS at 3 discrete frequency points obtained using frequency-domain NFFF transformation in the FDTD region is compared with the method of moments (MoM) solution in Fig. 5. It can be clearly observed that the higher order solution is closer to the MoM solution and outperforms the edge element based solution. In Fig. 6, the time-domain far-zone scattered electric field in the back-scattered direction using edge element and higher order basis functions are compared. The excitation is a φ-polarized differentiated Gaussian pulse and hence the θ- (cross) polarization component is weaker compared to the φ-component. Though not noticeable differences are observed in co-polarization component, the hybrid solution with higher order basis functions has a much lower cross-polarization component as should be the case for this geometry.

![Figure 5](image1.png)

Figure 5. Comparison of Bistatic RCS at 10GHz, 12 GHz and 15GHz obtained using the hybrid technique with the solution using traditional MoM. Hybrid-H0 and Hybrid-H1 refer to the case of edge element and higher order basis function respectively.

![Figure 6](image2.png)

Figure 6. Comparison of scattered time-domain far-zone electric field using edge element and higher order basis functions.

III. CONCLUSIONS

The 3D Hybrid FETD-FDTD method retains the inherent advantage of FETD method in modeling arbitrarily shaped structures along with the efficiency of FDTD method in modeling simple shapes and the PML. The method can be used for simulation of both electromagnetic scattering and radiation by antennas. Antenna feed with TEM excitation is incorporated in the FETD formulation enabling the extraction of modal reflection coefficient. Higher order basis functions are used in the FETD region for the better representation of the solution and enabling the use of coarser mesh. The use of hierarchical higher order basis functions in the finite element region enables us to revert back to lower order edge elements in the pyramidal and hexahedral elements, by which the hybridization of FDTD and FETD techniques remains unaffected. Results obtained using the method are promising and demonstrate its potential application to the analysis of radiation from broadband antennas and scattering from arbitrary structures. Improvement in accuracy with higher order basis functions in the FETD region was demonstrated highlighting the need for extending/developing the concepts of error estimation and adaptive refinement techniques to time domain methods.

REFERENCES


