ELECTROMAGNETIC DYADIC GREEN'S FUNCTIONS IN SPECTRAL DOMAIN FOR MULTILAYERED CYLINDERS

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Abstract—Dyadic Green’s function (DGF), as an electromagnetic response of dielectric medium or the field contributed due to a delta source, is quite useful to solve electromagnetic boundary-value problems. DGFs for multilayered structures are of particular interest to many engineers and scientists because of its better accuracy in modelling practical problems. Cylindrically multilayered medium is one of the most common structures used in practice, and the DGF for such a medium was recently published by Xiang and Lu [1]. However, it is found that there are some critical mistakes made and certain constraints assumed in the representation of the DGFs and their coefficients. The present paper serves as an amendment of [1] so as to avoid possible misleading of readers.

1. INTRODUCTION

The DGF technique is very powerful and elegant when used to solve boundary-value problems [2–5]. It has found two typical classes of applications. One of them is the employment of a DGF as an electromagnetic (EM) response of the dielectric medium involved so as to formulate the EM fields in a particular structure due to a known source of arbitrarily assumed distribution. The other is the application of a DGF as a kernel to derive the parameters matrices for the unknown coefficients of the basis function expansion of the unknown source current distribution in the Method of Moments.
Similar to the planar stratified and spherically layered media, a cylindrically multilayered medium is one of the most common structures in EM scattering and radiation problems. It is known that EM scattering problems associated with multilayered cylinder have been well-investigated, for example, the hybrid finite element method for coated cylinders [6] and the higher order boundary conditions for multilayered coated cylinders [7], the boundary condition matching for multilayered tree-trunk cylinders [8]. The EM scattering can be considered [9] as the EM radiation due to a source located at infinity. For the EM radiation due to a source in other regions of a multilayered structure, the formulae of EM scattering cannot be directly applied. In this sense, the EM radiation due to a source or several sources in arbitrarily multilayered cylinder is more complicated than the scattering.

To solve for the radiated fields in a cylinder of arbitrary multilayers, the DGF technique is an efficient way, as indicated earlier. The unbounded DGF in cylindrical coordinates was derived by Tai in 1971 [2, 1st Ed.] and the scattering DGFs for the regions in the presence of a conducting cylinder and a dielectric cylinder were also formulated in his classic book [2]. Later on, as an extension of the work, Yin and Wang [10] constructed the DGFs for a cylindrically multilayered chiral medium. However, the general coefficients were not obtained and the DGFs were written in a very tedious form in [10]. Furthermore, Li et al., [11] formulated a very compact and general form of DGFs and their coefficients for the cylindrically multilayered chiral medium. These DGFs and their coefficients reduce automatically to those for cylindrically multilayered achiral medium if the chirality parameter is assumed to be zero. However, this degeneration is somewhat difficult and time-consuming for those who want to make use of DGFs but fail to follow the complicated symbolic derivations in both [10] and [11]. Therefore, Xiang and Lu [1] recently published the DGF for cylindrically multilayered medium. The intention is good but there are some critical mistakes made and certain constraints assumed in the representation of the DGFs and their coefficients.

Prior to the publication of Xiang and Lu [1], the authors obtained in 1994 correct results and presented them in Dept. of Mathematics of NUS at the Interfaculty Seminar: Applications of Mathematics [12]. The critical mistakes and unnecessary constraints are realized [13] after we carefully read through the paper by Xiang and Lu [1]. Initially, it
was our thought that the mistakes could have arisen from typographic errors. However, on further investigation it was found from the fact that both their present form and possibly alternative form would not give the right answer.

To correct the mistakes in [1], one has to either generate very detailed comments based on the paper published by Xiang and Lu [1] or write a new paper. Since a major amendment needs to be made and it is not easy to follow the symbolic notations in that paper (e.g., the inconsistent use of vectors and matrices), we adopt the latter choice to re-write a paper instead. However, this paper is not a simply revised copy of what Xiang and Lu [1] published. In this paper, the complete and general coefficients of scattering DGFs are obtained not only for the general case of an arbitrary multilayered cylinder where source and observation points can be located anywhere, but also for two specific cases of cylindrically two-layered and three-layered media. These results are more general than those given for the axial symmetrical case \( n = 0 \) by Xiang and Lu. Besides, a unified procedure for obtaining these coefficients has been developed regardless of the location of both the source and the observation points, as compared with that published earlier.

2. FORMULATION OF THE BOUNDARY-VALUE PROBLEM

Since we use different symbolic notations, for readers' convenience and the paper's completeness, the fundamental formulae for the radiation problem are given briefly in this section. It is noted that all the vectors are written in bold and italic face while the matrix symbols are given in bold face only.

2.1 Fundamental Equations

Consider a medium of a cylindrically \( N \)-layered geometry shown in Fig. 1 and assume a time dependence \( \exp(-i\omega t) \) for the isotropic material throughout the paper. The transmitter with an arbitrary electric current distribution \( \mathbf{J}_s \) or an arbitrary magnetic current distribution \( \mathbf{M}_s \) is located in the \( s \)th (source) layer \( (s = 1, 2, \ldots, N) \), while the receiver lies in the \( f \)th (field) layer \( (f = 1, 2, \ldots, N) \) of the cylindrically \( N \)-layered medium.
For defining electromagnetic fields due to the electric and magnetic current sources, two types of dyadic Green’s functions, i.e., the electric type and the magnetic type of DGFs, are used. However, the electromagnetic types of the dyadic Green’s functions, $\mathbf{G}_e^{(fs)}(r, r')$ and $\mathbf{G}_m^{(fs)}(r, r')$, where the superscript $(fs)$ denotes the field and source points, and the subscripts $e$ and $m$ identify the electric and magnetic types of DGFs, are dual [2]. Therefore, only the electric type of dyadic Green’s function will be considered subsequently in this paper in order to avoid repetition, and the phrase “the electric type” will be omitted.

If the Green dyadics are known, the electromagnetic fields $E_f$ and $H_f$ in the $f$th layer due to an electric current $J_s$ in the $s$th layer can be obtained in terms of Green dyadic $\mathbf{G}_e^{(fs)}(r, r')$ as follows:

$$E_f(r) = i\omega \mu_f \iint_{V_s} \mathbf{G}_e^{(fs)}(r, r') \cdot J_s(r') dV', \quad (1a)$$

$$H_f(r) = \iint_{V_s} \left[ \mathbf{\nabla} \times \mathbf{G}_e^{(fs)}(r, r') \right] \cdot J_s(r') dV', \quad (1b)$$

where $V_s$ identifies the volume occupied by the sources in the $s$th layer, and $\mu_f$ represents the permeability of the medium. Vectors are written in the bold and italic faces throughout the paper while the matrices are written in the bold face only.
It is shown [2] that the dyadic Green's function satisfies the following boundary conditions at the cylindrical interfaces $r = a_j$ ($j = 1, 2, \ldots, N - 1$):

$$\hat{r} \times \overline{G}_e^{(fs)} = \hat{r} \times \overline{G}_e^{[(f+1)s]},$$

$$\frac{1}{\mu_f} \hat{r} \times \nabla \times \overline{G}_e^{(fs)} = \frac{1}{\mu_{f+1}} \hat{r} \times \nabla \times \overline{G}_e^{[(f+1)s]}.$$

### 2.2 Unbounded Dyadic Green's Function

In order to obtain the solution of $\overline{G}_e^{(fs)}(r, r')$, the method of scattering superposition is used in this paper. Thus, the dyadic Green's function can be separated into two parts, i.e., the unbounded DGF and the scattering DGF, as follows [2]:

$$\overline{G}_e^{(fs)}(r, r') = \overline{G}_{0e}(r, r') \delta^s_f + \overline{G}_{es}^{(fs)}(r, r'),$$

where the subscript $s$ stands for the scattering DGF, and $\delta^s_f$ denotes the Kronecker delta. The unbounded dyadic Green's function $\overline{G}_{0e}(r, r')$ represents the contribution of the direct waves from radiation sources in an unbounded medium, and the scattering dyadic Green's function $\overline{G}_{es}^{(fs)}(r, r')$ describes an additional contribution of the multiple reflection and transmission waves from the cylindrical interfaces of cylindrically stratified media.

This unbounded Green's dyadic $\overline{G}_{0e}(r, r')$ was given [2] for $r \geq r'$ by

$$\overline{G}_{0e}(r, r') = -\frac{\hat{r} \hat{r} \delta(r - r')}{k_s^2} + \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{(2 - \delta_0^n)}{\eta_s^2}$$

$$\left\{ M_{\phi \phi}^{(1)}(h) M_{\phi \phi}^{(1)}(-h) + N_{\phi \phi}^{(1)}(h) N_{\phi \phi}^{(1)}(-h) \right\},$$

where the prime denotes the coordinates $(r', \phi', z')$ of the current source $J_f$ and the superscript (1) of the vector wave functions denotes that the third-type cylindrical Bessel function or the first-type cylindrical Hankel function $H_n^{(1)}(\eta_f r)$ is used in the following expressions.
of the cylindrical wave vector functions for the out-going waves:

\[
M_{\phi}^{n\eta_f}(\hat{h}) = \nabla \times \left[ Z_n(\eta_f r) \cos(n\phi)e^{ihz}\hat{z} \right], \quad (5a)
\]

\[
N_{\phi}^{n\eta_f}(\hat{h}) = \frac{1}{\sqrt{h^2 + \eta_f^2}} \nabla \times \nabla \times \left[ Z_n(\eta_f r) \cos(n\phi)e^{ihz}\hat{z} \right], \quad (5b)
\]

where \( n \) and \( \eta_f \) are the eigenvalues, and \( Z_n(\eta_f r) \) usually takes the form of the first-type cylindrical Bessel function \( J_n(\eta_f r) \), and denotes the cylindrical Bessel function of \( n \)-order. The eigenvalue, \( \eta_f \), and the propagation constant, \( k_f \), in the \( f \)th layer, satisfy the following relation:

\[
h^2 = (k_f)^2 - (\eta_f)^2,
\]

\[
k_f^2 = \omega^2 \mu_f \varepsilon_f \left( 1 + \frac{i\sigma_f}{\omega \varepsilon_f} \right),
\]

where \( \varepsilon_f \) and \( \sigma_f \) denote the permittivity and conductivity of the medium, respectively.

2.3 Scattering Dyadic Green’s Function

In terms of cylindrical vector wave functions shown in (5a) and (5b), the scattering DGF can be constructed. Yin and Wang [10] expressed the DGF in a quite tedious form for a multilayered chiral medium in three special cases, i.e., the source located in the first, the intermediate and the last layers. Later, Li et al., represented the DGF in a greatly compact form by taking the advantage of the Kronecker delta. In a similar fashion, Xiang and Lu obtained the DGF in terms of matrix form and published [1] the DGF recently for the cylindrically multi-layered achiral media. However, the DGF was incorrectly constructed since the boundary conditions on the cylindrical interfaces cannot be satisfied. The correct form of the DGF for the multilayered cylinder is given by:
\[ G_{es}^{(fs)}(r, r') = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{(2 - \delta_n^0)}{\eta_n^2} \]

\[ \cdot \left\{ (1 - \delta_j^N) M_{\delta_{n\eta f}}^{(1)}(h) \left[ (1 - \delta_s^1) C_{1H}^{fs} M'_{\delta_{n\eta s}}(-h) \right] + (1 - \delta_s^N) C_{1H}^{fs} M'_{\delta_{n\eta s}}(-h) \right\} \]

\[ + (1 - \delta_j^N) N_{\delta_{n\eta f}}^{(1)}(h) \left[ (1 - \delta_s^1) C_{1V}^{fs} N'_{\delta_{n\eta s}}(-h) \right] \]

\[ + (1 - \delta_s^N) C_{1V}^{fs} N'_{\delta_{n\eta s}}(-h) \]

\[ + (1 - \delta_j^N) N_{\delta_{n\eta f}}^{(1)}(h) \left[ (1 - \delta_s^1) C_{2H}^{fs} M'_{\delta_{n\eta s}}(-h) \right] \]

\[ + (1 - \delta_j^N) C_{2H}^{fs} M'_{\delta_{n\eta s}}(-h) \]

\[ + (1 - \delta_s^N) C_{2V}^{fs} M'_{\delta_{n\eta s}}(-h) \]

\[ + (1 - \delta_s^N) C_{2V}^{fs} N'_{\delta_{n\eta s}}(-h) \]

\[ + (1 - \delta_j^N) M_{\delta_{n\eta f}}^{(1)}(h) \left[ (1 - \delta_s^1) C_{3H}^{fs} M'_{\delta_{n\eta s}}(-h) \right] \]

\[ + (1 - \delta_j^N) C_{3H}^{fs} M'_{\delta_{n\eta s}}(-h) \]

\[ + (1 - \delta_s^N) C_{3V}^{fs} N'_{\delta_{n\eta s}}(-h) \]

\[ + (1 - \delta_j^N) N_{\delta_{n\eta f}}^{(1)}(h) \left[ (1 - \delta_s^1) C_{4H}^{fs} M'_{\delta_{n\eta s}}(-h) \right] \]

\[ + (1 - \delta_s^N) C_{4H}^{fs} M'_{\delta_{n\eta s}}(-h) \]
+ (1 - \delta_f^1)M_{e^s_{\eta_{nf}}}(h) \left[(1 - \delta_s^1)C_{4V}^{fs}N_{e^s_{\eta_{ns}}}(-h) + (1 - \delta_s^N)C_{4V}^{fst}N_{e^s_{\eta_{ns}}}^{(1)}(-h) \right], \quad (6)

where, \( f \) and \( s \) stand for the \( f \)th (field) layer and the \( s \)th (source) layer, respectively. The superscript \( N \) of the Kronecker delta \( \delta_s^N \) represents the layer number of the cylindrical medium, and \( C_{mP}^{fs} \) \((m = 1, 2, 3 \text{ and } 4, \text{ and } P \text{ denotes } H \text{ for TE waves or } V \text{ for TM waves})\) are the coefficients of the scattered Green dyadics to be solved.

3. RECURRENCE MATRICES

Because of the wrong formulation of the DGF in [1], the scattering coefficients of the DGFs were improperly derived as well. Here in this paper, we will re-formulate these coefficients.

To solve for the unknown coefficients in (6), the boundary conditions satisfied by the Green dyadic in the cylindrically multilayered medium are used. For succinct formulation, the following terse matrix equation satisfied by the coefficient matrices can be obtained from the boundary conditions, Eqs. (2a), (2b), for the TE and TM waves (notated \( H \) and \( V \), respectively):

\[
F_{(f+1)f}^{(H,V)} \cdot \left[C_{(f+1)s}^{(H,V)} + \delta_{f+1}^s A_1 \right] = F_{ff}^{(H,V)} \cdot \left[C_{fs}^{(H,V)} + \delta_{f}^s A_2 \right], \quad (7)
\]

where the coefficient matrix is

\[
C_{fs}^{(H,V)} = \begin{bmatrix}
(1 - \delta_f^N)(1 - \delta_s^1)C_{1(H,V)}^{fs} & (1 - \delta_f^N)(1 - \delta_s^N)C_{1(H,V)}^{fst} \\
(1 - \delta_f^N)(1 - \delta_s^1)C_{2(H,V)}^{fs} & (1 - \delta_f^N)(1 - \delta_s^N)C_{2(H,V)}^{fst} \\
(1 - \delta_f^1)(1 - \delta_s^1)C_{3(H,V)}^{fs} & (1 - \delta_f^1)(1 - \delta_s^N)C_{3(H,V)}^{fst} \\
(1 - \delta_f^1)(1 - \delta_s^N)C_{4(H,V)}^{fs} & (1 - \delta_f^1)(1 - \delta_s^N)C_{4(H,V)}^{fst}
\end{bmatrix} \quad ; \quad (8)
\]

the parameter matrices are

\[
A_1 = \begin{bmatrix}
1 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0
\end{bmatrix} \quad ; \quad (9)
\]
and the transmission matrices are given by

\[
F_{jm} = \begin{bmatrix}
\frac{\partial H_n^{(1)}(\eta_j a_m)}{a_m} & \frac{\partial J_n(\eta_j a_m)}{a_m} & \frac{\partial H_n^{(1)}(\eta_j a_m)}{a_m} & \frac{\partial J_n(\eta_j a_m)}{a_m} \\
0 & 0 & 0 & 0 \\
\frac{\partial H_n^{(1)}(\eta_j a_m)}{a_m} & \frac{\partial J_n(\eta_j a_m)}{a_m} & \frac{\partial H_n^{(1)}(\eta_j a_m)}{a_m} & \frac{\partial J_n(\eta_j a_m)}{a_m} \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad (10a)
\]

\[
F_{jm} = \begin{bmatrix}
\frac{\partial H_n^{(1)}(\eta_j a_m)}{a_m} & \frac{\partial J_n(\eta_j a_m)}{a_m} & \frac{\partial H_n^{(1)}(\eta_j a_m)}{a_m} & \frac{\partial J_n(\eta_j a_m)}{a_m} \\
0 & 0 & 0 & 0 \\
\frac{\partial H_n^{(1)}(\eta_j a_m)}{a_m} & \frac{\partial J_n(\eta_j a_m)}{a_m} & \frac{\partial H_n^{(1)}(\eta_j a_m)}{a_m} & \frac{\partial J_n(\eta_j a_m)}{a_m} \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad (10b)
\]

\[j = 1, 2, \ldots, N, \quad m = 1, 2, \ldots, N - 1.\]

In Eqs. (10a) and (10b), the following inter-parameters have been used to simplify the complicated algebraic expression:

\[\tau_j = \sqrt{\frac{\varepsilon_j}{\mu_j}}, \quad \zeta_j = \frac{i\hbar n}{k_j}, \quad \varrho_j = \frac{(\eta_j)^2}{k_j}. \quad (11)\]

Let the following transmission matrix be given by

\[T_f^{(H,V)} = \left[\mathcal{R}_f^{(H,V)}\right]_{4 \times 4} = \left[F_{(f+1)f}^{(H,V)}\right]^{-1} \cdot \left[F_{ff}^{(H,V)}\right], \quad (12)\]

where \(\left[F_{(f+1)f}^{(H,V)}\right]^{-1}\) is the inverse matrix of \(F_{(f+1)f}^{(H,V)}\) inverted by making use of Gaussian-Jordan elimination. Thus, the linear equation (7) can be rewritten into the following recurrence form:

\[C_{(f+1)s}^{(H,V)} = T_f^{(H,V)} \cdot \left[C_{fs}^{(H,V)} + \delta_f^s A_2\right] - \delta_{f+1}^s A_1. \quad (13)\]

4. ANALYTIC EXPRESSION OF THE COEFFICIENTS

Thus, all that remains to be done now is to solve the \(N\) matrix equations derived from the recurrence relation (13) for the unknown coefficients \(C_1^{(H,V)}, C_2^{(H,V)}, \ldots, C_{N_s}^{(H,V)}\). The procedure for deriving the coefficients of the scattering DGFs has been introduced in [11] where
three cases, the sources are located in the outer layer, the intermediate layers, and the inner layer, have to be separately considered. In this paper, this procedure is unified so that the solution obtained is valid for all the cases regardless of the position of the source points.

From Eq. (13), we obtain a recurrence relation given as follows when the current source is located anywhere in the media:

\[
C_{fs}^{(H,V)} = T_{f-1}^{(H,V)} \cdots T_{s}^{(H,V)} \cdot \left[ T_{s-1}^{(H,V)} \cdots T_{1}^{(H,V)} \cdot C_{1s}^{(H,V)} + (1 - \delta_s^N)u(f-s-1)A_2 - (1 - \delta_s^1)u(f-s)A_1 \right], \tag{14}
\]

where \( u(x) = 1 \) for \( x \geq 0 \), and \( 0 \) for \( x < 0 \) denotes the step function.

By letting \( f = N \) in the equation (14), we have a matrix equation containing the coefficients only for the first layer and the last layer. Solving the matrix equation obtained, we can derive the coefficients for the first layer and the last layer. Let us define that

\[
T_{K}^{(H,V)} = \left[ T_{ij}^{K(H,V)} \right]_{4 \times 4} = \left[ T_{N-1}^{(H,V)} \right] \left[ T_{N-2}^{(H,V)} \right] \cdots \left[ T_{K+1}^{(H,V)} \right] \left[ T_{K}^{(H,V)} \right]. \tag{15}
\]

The coefficients for the first layer is determined from the following matrix equation:

\[
\begin{pmatrix}
(1 - \delta_1^1)C_{11}^{1s} & (1 - \delta_s^N)C_{11}^{1s} \\
(1 - \delta_s^1)C_{12}^{1s} & (1 - \delta_s^N)C_{12}^{1s}
\end{pmatrix}
= 
\begin{pmatrix}
T_{11}^{1(H,V)} & T_{12}^{1(H,V)} \\
T_{21}^{1(H,V)} & T_{22}^{1(H,V)}
\end{pmatrix}
- 
\begin{pmatrix}
T_{13}^{s(H,V)} & T_{14}^{s(H,V)} \\
T_{23}^{s(H,V)} & T_{24}^{s(H,V)}
\end{pmatrix}
\cdot
\begin{pmatrix}
0 & (1 - \delta_s^N) \\
0 & 0
\end{pmatrix}
\cdot
\begin{pmatrix}
(1 - \delta_1^1) & 0 \\
0 & 0
\end{pmatrix}
\tag{16}
\]

In terms of the coefficients for the first layer in Eq. (16), the coefficients of the last layer are given by the matrix equation as follows:
Substituting the coefficients in (16) into (14), we obtain the rest of the DGF’s coefficients.

So far, the whole set of the DGF’s coefficients has been obtained regardless of whether the electric current source is located inside or outside the cylindrically multilayered cylinder. They are expressed rigorously here in terms of compact recurrent matrices. To show how specific Green dyadics for simple geometries are obtained from the general results, further reduction has been made for formulating the dyadic Green’s functions for a cylinder, as an example.

5. APPLICATION I: A TWO-LAYERED CYLINDER

A cylinder can be considered as a two-layered medium, i.e., \( N = 2 \). The scattering dyadic Green’s function and its coefficients can be expressed in the order of the layer below. In Ref. [1], the results are valid for the lowest but non-dominant mode \( n = 0 \) corresponding to the axial symmetry only. In this paper, the results given here are valid for all the modes.

5.1 Current Source outside Cylinder

When a current source is located outside the cylinder (i.e., \( s = 1 \)), the dyadic Green’s function can be written as:

\[
\begin{bmatrix}
(1 - \delta_s^1) C_{3(1)(1)}^{N1} & (1 - \delta_s^N) C_{3(1)(1)}^{N1} \\
(1 - \delta_s^1) C_{4(1)(1)}^{N1} & (1 - \delta_s^N) C_{4(1)(1)}^{N1}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
T_{31}^{1(1)}(H,V) & T_{32}^{1(1)}(H,V) \\
T_{41}^{1(1)}(H,V) & T_{42}^{1(1)}(H,V)
\end{bmatrix}
\begin{bmatrix}
C_{1(1)(1)}^{1s} & C_{1(1)(1)}^{1s} \\
C_{2(1)(1)}^{1s} & C_{2(1)(1)}^{1s}
\end{bmatrix}
\]

\[
- \begin{bmatrix}
T_{31}^{s(H,V)} & T_{32}^{s(H,V)} \\
T_{41}^{s(H,V)} & T_{42}^{s(H,V)}
\end{bmatrix}
\begin{bmatrix}
(1 - \delta_s^1) & 0 \\
0 & 0
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
T_{33}^{s(H,V)} & T_{34}^{s(H,V)} \\
T_{43}^{s(H,V)} & T_{44}^{s(H,V)}
\end{bmatrix}
\begin{bmatrix}
0 & (1 - \delta_s^N) \\
0 & 0
\end{bmatrix}.
\]
for \( f = 1 \)

\[
\bar{G}_{es}^{(1)}(r, r') = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{2 - \delta_n^0}{\eta_1^2} \\
\cdot \left[ C_{1\eta_1}^{11} M_{e,\eta_1}^{(1)}(h) M_{e,\eta_1}^{(1)}(-h) + C_{1\eta_1}^{11} N_{e,\eta_1}^{(1)}(h) \\
\cdot N_{e,\eta_1}^{(1)}(-h) + C_{2\eta_1}^{11} M_{e,\eta_1}^{(1)}(-h) \right].
\]

(18a)

and for \( f = 2 \)

\[
\bar{G}_{es}^{(21)}(r, r') = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{2 - \delta_n^0}{\eta_1^2} \\
\cdot \left[ C_{3\eta_2}^{21} M_{e,\eta_2}^{(1)}(h) M_{e,\eta_2}^{(1)}(-h) + C_{3\eta_2}^{21} N_{e,\eta_2}^{(1)}(h) \\
\cdot N_{e,\eta_2}^{(1)}(-h) + C_{4\eta_2}^{21} M_{e,\eta_2}^{(1)}(-h) \right].
\]

(18b)

According to Eq. (14), we have the following recurrence relation for \( s = 1 \):

\[
C_{f1}^{(H,V)} = \left[ T_{f-1}^{(H,V)} \right] \cdots T_1^{(H,V)} \cdot \left[ C_{11}^{(H,V)} + A_2 \right].
\]

(19)

By letting \( f = 2 \) in (19), a recurrence relation satisfied by the coefficient matrices for the inner and outer regions of the cylinder can be obtained. The unknown coefficients for the outer region can be determined from the matrix equation as follows:

\[
\begin{bmatrix}
C_{11}^{(H,V)} \\
C_{21}^{(H,V)}
\end{bmatrix} = - \begin{bmatrix}
T_{11}^{(H,V)} & T_{12}^{(H,V)} \\
T_{21}^{(H,V)} & T_{22}^{(H,V)}
\end{bmatrix}^{-1} \begin{bmatrix}
T_{13}^{(H,V)} \\
T_{23}^{(H,V)}
\end{bmatrix}.
\]

(20)

Using the coefficients in (20), we may derive the coefficients for the inner region of the cylinder as follows:

\[
\begin{bmatrix}
C_{31}^{(H,V)} \\
C_{41}^{(H,V)}
\end{bmatrix} = \begin{bmatrix}
T_{31}^{(H,V)} & T_{32}^{(H,V)} \\
T_{41}^{(H,V)} & T_{42}^{(H,V)}
\end{bmatrix} \begin{bmatrix}
C_{11}^{(H,V)} \\
C_{21}^{(H,V)}
\end{bmatrix} + \begin{bmatrix}
T_{33}^{(H,V)} \\
T_{43}^{(H,V)}
\end{bmatrix}.
\]

(21)
Thus, all the coefficients of the scattering dyadic Green’s function for the two layers are obtained when the current source is present outside of the cylinder. Comparison of the coefficients presented here with those already obtained by Tai [2] shows a good agreement, demonstrating the applicability of the general coefficients.

5.2 Current Source inside Cylinder

When the current source is located inside the cylinder, the dyadic Green’s function can be given below:

\[
\mathcal{G}^{(12)}_{es}(r, r') = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{(2 - \delta_n^0)}{\eta_2^n} \left[ C_{1H}^{12} M_{e\eta_1}^{(1)} (h) M_{e\eta_2}^{(1)} (-h) + C_{1V}^{12} N_{o\eta_1}^{(1)} (h) \right.
\]

\[
\cdot N_{e\eta_2}^{o\eta_2} (-h) + C_{2H}^{12} M_{o\eta_1}^{(1)} (h) M_{e\eta_2}^{(1)} (-h)
\]

\[
+ C_{2V}^{12} M_{e\eta_1}^{(1)} (h) N_{e\eta_2}^{o\eta_2} (-h) \right], \quad (22a)
\]

and for \( f = 2 \)

\[
\mathcal{G}^{(22)}_{es}(r, r') = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{(2 - \delta_n^0)}{\eta_2^n} \left[ C_{3H}^{22} M_{o\eta_2}^{e\eta_2} (h) M_{e\eta_2}^{(1)} (-h) + C_{3V}^{22} N_{o\eta_2}^{e\eta_2} (h) \right.
\]

\[
\cdot N_{e\eta_2}^{o\eta_2} (-h) + C_{4H}^{22} M_{o\eta_2}^{e\eta_2} (h) M_{e\eta_2}^{(1)} (-h)
\]

\[
+ C_{4V}^{22} M_{e\eta_2}^{o\eta_2} (h) N_{e\eta_2}^{o\eta_2} (-h) \right]. \quad (22b)
\]

Similarly, we may derive the following recurrence equation:

\[
C_{fN}^{(H,V)} = \left[ T_{(H,V)}^{(f)} \right]^{-1} T_{(H,V)}^{(1)} C_{1N}^{(H,V)} - u(f - N)A_1. \quad (23)
\]

Thus, the coefficients of the scattering Green dyadics can be obtained from Eq. (23) by substituting \( f = N \) into the equation. The results
for the first layer are given by:

\[
\begin{bmatrix}
C_{12}^{12(H,V)} \\
C_{22}^{12(H,V)}
\end{bmatrix} =
\begin{bmatrix}
T_{11}^{1(H,V)} & T_{12}^{1(H,V)} \\
T_{21}^{1(H,V)} & T_{22}^{1(H,V)}
\end{bmatrix}^{-1} \begin{bmatrix}
1 \\
0
\end{bmatrix}.
\] (24)

By substituting the coefficients for the first layer shown in Eq. (24), the coefficients for the inner region can be obtained by

\[
\begin{bmatrix}
C_{33}^{22(H,V)} \\
C_{44}^{22(H,V)}
\end{bmatrix} =
\begin{bmatrix}
T_{31}^{1(H,V)} & T_{32}^{1(H,V)} \\
T_{41}^{1(H,V)} & T_{42}^{1(H,V)}
\end{bmatrix} \begin{bmatrix}
C_{12}^{11(H,V)} \\
C_{22}^{12(H,V)}
\end{bmatrix}.
\] (25)

As an example of the applications of the method presented, this section shows how the results for a multilayered cylinder are degenerated to those for a two-layered cylindrical medium. Reductions of the general coefficients can, however, also be made for other specific media of simple layered geometries such as the following three-layered geometry.

6. APPLICATION II: A THREE-LAYERED CYLINDER

In the case of the three-layered medium, the dyadic Green's function and the coefficients of the scattering dyadic Green's function can also be represented by letting \( N = 3 \), according to the order of the layer as shown below. In a similar fashion to the application I, the results given in this section are also valid for all the modes.

6.1 Current Source in the First Layer

When the current source is located in the first layer \((s = 1)\), the dyadic Green's function can be written as:

for \( f = 1 \)

\[
\bar{G}_{es}^{(1)}(r, r') = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{(2 - \delta_n^0)}{n_l} \\
\cdot \left[ C_{11}^{11}(h)M_{e}^{(1)}_{\phi n_1}(h)M_{e}^{(1)}_{\phi n_1}(-h) + C_{11}^{11}N_{e}^{(1)}_{\phi n_1}(h) \\
+ C_{21}^{11}N_{e}^{(1)}_{\phi n_1}(-h) \right],
\] (26a)
for \( f = 2 \)

\[
\overline{G}_{es}^{(21)}(r, r') = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{(2 - \delta_n^0)}{\eta_1^2} \left[ C_{1H}^{21} M_{e_{\eta_2}}^{(1)}(h) M_{e_{\eta_1}}^{(1)}(-h) + C_{1V}^{21} N_{e_{\eta_2}}^{(1)}(h) \right.
\]
\[
\cdot N_{e_{\eta_1}}^{(1)}(-h) + C_{2V}^{21} M_{e_{\eta_2}}^{(1)}(h) M_{e_{\eta_1}}^{(1)}(-h) + C_{3H}^{21} M_{e_{\eta_2}}^{(1)}(h) M_{e_{\eta_1}}^{(1)}(-h)
\]
\[
+ C_{3V}^{21} M_{e_{\eta_2}}^{(1)}(h) N_{e_{\eta_1}}^{(1)}(-h) + C_{4H}^{21} N_{e_{\eta_2}}^{(1)}(h) M_{e_{\eta_1}}^{(1)}(-h) + C_{4V}^{21} M_{e_{\eta_2}}^{(1)}(h) N_{e_{\eta_1}}^{(1)}(-h) \left. \right] ,
\]
\[\text{(26b)}\]

and for \( f = 3 \)

\[
\overline{G}_{es}^{(31)}(r, r') = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{(2 - \delta_n^0)}{\eta_1^2} \left[ C_{3H}^{31} M_{e_{\eta_3}}^{(1)}(h) M_{e_{\eta_1}}^{(1)}(-h) + C_{3V}^{31} N_{e_{\eta_3}}^{(1)}(h) \right.
\]
\[
\cdot N_{e_{\eta_1}}^{(1)}(-h) + C_{4H}^{31} M_{e_{\eta_3}}^{(1)}(h) M_{e_{\eta_1}}^{(1)}(-h) + C_{4V}^{31} M_{e_{\eta_3}}^{(1)}(h) N_{e_{\eta_1}}^{(1)}(-h)
\]
\[
+ C_{4V}^{31} M_{e_{\eta_3}}^{(1)}(h) N_{e_{\eta_1}}^{(1)}(-h) \left. \right] .
\]
\[\text{(26c)}\]

From Eq. (15), we have

\[
T_{(H,V)}^{(1)} = \begin{bmatrix}
T_{11}^{1(H,V)} & T_{12}^{1(H,V)} & T_{13}^{1(H,V)} & T_{14}^{1(H,V)} \\
T_{21}^{1(H,V)} & T_{22}^{1(H,V)} & T_{23}^{1(H,V)} & T_{24}^{1(H,V)} \\
T_{31}^{1(H,V)} & T_{32}^{1(H,V)} & T_{33}^{1(H,V)} & T_{34}^{1(H,V)} \\
T_{41}^{1(H,V)} & T_{42}^{1(H,V)} & T_{43}^{1(H,V)} & T_{44}^{1(H,V)} 
\end{bmatrix}.
\]
\[\text{(27a)}\]
\[ T^{(2)}_{(H,V)} = \begin{bmatrix}
R_{1(11)}^{(H,V)} & R_{1(12)}^{(H,V)} & R_{1(13)}^{(H,V)} & R_{1(14)}^{(H,V)} \\
R_{2(11)}^{(H,V)} & R_{2(12)}^{(H,V)} & R_{2(13)}^{(H,V)} & R_{2(14)}^{(H,V)} \\
R_{3(11)}^{(H,V)} & R_{3(12)}^{(H,V)} & R_{3(13)}^{(H,V)} & R_{3(14)}^{(H,V)} \\
R_{4(11)}^{(H,V)} & R_{4(12)}^{(H,V)} & R_{4(13)}^{(H,V)} & R_{4(14)}^{(H,V)}
\end{bmatrix}, \quad (27b)\]

where the elements in Eq. (27a) are shown to be

\[ T^{(1)(H,V)}_{11} = R_{1(11)}^{(H,V)} R_{2(11)}^{(H,V)} + R_{1(21)}^{(H,V)} R_{2(12)}^{(H,V)} + R_{1(31)}^{(H,V)} R_{2(13)}^{(H,V)} + R_{1(41)}^{(H,V)} R_{2(14)}^{(H,V)}, \quad (28a)\]

\[ T^{(1)(H,V)}_{12} = R_{1(12)}^{(H,V)} R_{2(11)}^{(H,V)} + R_{1(22)}^{(H,V)} R_{2(12)}^{(H,V)} + R_{1(32)}^{(H,V)} R_{2(13)}^{(H,V)} + R_{1(42)}^{(H,V)} R_{2(14)}^{(H,V)}, \quad (28b)\]

\[ T^{(1)(H,V)}_{13} = R_{1(13)}^{(H,V)} R_{2(11)}^{(H,V)} + R_{1(23)}^{(H,V)} R_{2(12)}^{(H,V)} + R_{1(33)}^{(H,V)} R_{2(13)}^{(H,V)} + R_{1(43)}^{(H,V)} R_{2(14)}^{(H,V)}, \quad (28c)\]

\[ T^{(1)(H,V)}_{14} = R_{1(14)}^{(H,V)} R_{2(11)}^{(H,V)} + R_{1(24)}^{(H,V)} R_{2(12)}^{(H,V)} + R_{1(34)}^{(H,V)} R_{2(13)}^{(H,V)} + R_{1(44)}^{(H,V)} R_{2(14)}^{(H,V)}, \quad (28d)\]

\[ T^{(1)(H,V)}_{21} = R_{1(11)}^{(H,V)} R_{2(21)}^{(H,V)} + R_{1(21)}^{(H,V)} R_{2(22)}^{(H,V)} + R_{1(31)}^{(H,V)} R_{2(23)}^{(H,V)} + R_{1(41)}^{(H,V)} R_{2(24)}^{(H,V)}, \quad (28e)\]

\[ T^{(1)(H,V)}_{22} = R_{1(12)}^{(H,V)} R_{2(21)}^{(H,V)} + R_{1(22)}^{(H,V)} R_{2(22)}^{(H,V)} + R_{1(32)}^{(H,V)} R_{2(23)}^{(H,V)} + R_{1(42)}^{(H,V)} R_{2(24)}^{(H,V)}, \quad (28f)\]

\[ T^{(1)(H,V)}_{23} = R_{1(13)}^{(H,V)} R_{2(21)}^{(H,V)} + R_{1(23)}^{(H,V)} R_{2(22)}^{(H,V)} + R_{1(33)}^{(H,V)} R_{2(23)}^{(H,V)} + R_{1(43)}^{(H,V)} R_{2(24)}^{(H,V)}, \quad (28g)\]

\[ T^{(1)(H,V)}_{24} = R_{1(14)}^{(H,V)} R_{2(21)}^{(H,V)} + R_{1(24)}^{(H,V)} R_{2(22)}^{(H,V)} + R_{1(34)}^{(H,V)} R_{2(23)}^{(H,V)} + R_{1(44)}^{(H,V)} R_{2(24)}^{(H,V)}, \quad (28h)\]

\[ T^{(1)(H,V)}_{31} = R_{1(11)}^{(H,V)} R_{2(31)}^{(H,V)} + R_{1(21)}^{(H,V)} R_{2(32)}^{(H,V)} + R_{1(31)}^{(H,V)} R_{2(33)}^{(H,V)} + R_{1(41)}^{(H,V)} R_{2(34)}^{(H,V)}, \quad (28i)\]
Thus, the coefficients of scattered DGF can be derived as:

\[ C_{11}^{11} = \frac{1}{D_3} \left( \mathcal{T}_{12}^{1(H,V)} \mathcal{T}_{23}^{1(H,V)} - \mathcal{T}_{22}^{1(H,V)} \mathcal{T}_{13}^{1(H,V)} \right), \]  
\[ C_{21}^{11} = \frac{1}{D_3} \left( \mathcal{T}_{21}^{1(H,V)} \mathcal{T}_{13}^{1(H,V)} - \mathcal{T}_{11}^{1(H,V)} \mathcal{T}_{23}^{1(H,V)} \right), \]  
\[ C_{31}^{11} = \mathcal{R}_{1(11)}^{(H,V)} \mathcal{C}_{1(1)}^{11} + \mathcal{R}_{1(12)}^{(H,V)} \mathcal{C}_{2(1)}^{11} + \mathcal{R}_{1(13)}^{(H,V)}, \]  
\[ C_{22}^{11} = \mathcal{R}_{1(21)}^{(H,V)} \mathcal{C}_{1(1)}^{11} + \mathcal{R}_{1(22)}^{(H,V)} \mathcal{C}_{2(2)}^{11} + \mathcal{R}_{1(23)}^{(H,V)}, \]  
\[ C_{32}^{11} = \mathcal{R}_{1(31)}^{(H,V)} \mathcal{C}_{1(1)}^{11} + \mathcal{R}_{1(32)}^{(H,V)} \mathcal{C}_{2(2)}^{11} + \mathcal{R}_{1(33)}^{(H,V)}, \]  
\[ C_{23}^{11} = \mathcal{R}_{1(21)}^{(H,V)} \mathcal{C}_{1(2)}^{11} + \mathcal{R}_{1(22)}^{(H,V)} \mathcal{C}_{2(2)}^{11} + \mathcal{R}_{1(23)}^{(H,V)}, \]  
\[ C_{33}^{11} = \mathcal{T}_{31}^{1(H,V)} \mathcal{C}_{1(1)}^{11} + \mathcal{T}_{32}^{1(H,V)} \mathcal{C}_{2(2)}^{11} + \mathcal{T}_{33}^{1(H,V)}, \]  
\[ C_{34}^{11} = \mathcal{T}_{41}^{1(H,V)} \mathcal{C}_{1(1)}^{11} + \mathcal{T}_{42}^{1(H,V)} \mathcal{C}_{2(2)}^{11} + \mathcal{T}_{43}^{1(H,V)} \mathcal{C}_{2(3)}^{11} + \mathcal{T}_{44}^{1(H,V)} \mathcal{C}_{2(4)}^{11}, \]  
\[ C_{43}^{11} = \mathcal{T}_{41}^{1(H,V)} \mathcal{C}_{1(1)}^{11} + \mathcal{T}_{42}^{1(H,V)} \mathcal{C}_{2(2)}^{11} + \mathcal{T}_{43}^{1(H,V)} \mathcal{C}_{2(3)}^{11} + \mathcal{T}_{44}^{1(H,V)} \mathcal{C}_{2(4)}^{11}. \]
where

$$D_3 = T_{11}^{1(H,V)} T_{22}^{1(H,V)} - T_{12}^{1(H,V)} T_{21}^{1(H,V)}.$$

### 6.2 Current Source in the Medium Layer

In the case of the current source located in the medium layer, the dyadic Green’s function is constructed as follows: for $f = 1$

$$\overline{G^{(12)}_{es}(r, r')} = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{(2 - \delta_{n}^{0})}{\eta_2} \left\{ M^{(1)}_{e_{\eta_1}}(h) \left[ C_{1H}^{1} M_{e_{\eta_2}}^{1}(-h) + C_{1H}^{12} M_{e_{\eta_2}}^{1}(-h) \right] \\
+ N^{(1)}_{e_{\eta_1}}(h) \left[ C_{1V}^{1} N_{e_{\eta_2}}^{1}(-h) + C_{1V}^{12} N_{e_{\eta_2}}^{1}(-h) \right] \\
+ N^{(1)}_{o_{\eta_1}}(h) \left[ C_{2H}^{1} M_{o_{\eta_2}}^{1}(-h) + C_{2H}^{12} M_{o_{\eta_2}}^{1}(-h) \right] \\
+ M^{(1)}_{o_{\eta_2}}(h) \left[ C_{2V}^{1} N_{o_{\eta_2}}^{1}(-h) + C_{2V}^{12} N_{o_{\eta_2}}^{1}(-h) \right] \right\}, \quad (30a)$$

for $f = 2$

$$\overline{G^{(22)}_{es}(r, r')} = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{(2 - \delta_{n}^{0})}{\eta_2} \left\{ M^{(1)}_{o_{\eta_2}}(h) \left[ C_{1H}^{22} M_{e_{\eta_2}}^{1}(-h) + C_{1H}^{22} M_{e_{\eta_2}}^{1}(-h) \right] \\
+ N^{(1)}_{e_{\eta_2}}(h) \left[ C_{1V}^{22} N_{e_{\eta_2}}^{1}(-h) + C_{1V}^{22} N_{e_{\eta_2}}^{1}(-h) \right] \\
+ N^{(1)}_{o_{\eta_2}}(h) \left[ C_{2H}^{22} M_{o_{\eta_2}}^{1}(-h) + C_{2H}^{22} M_{o_{\eta_2}}^{1}(-h) \right] \\
+ M^{(1)}_{e_{\eta_2}}(h) \left[ C_{2V}^{22} N_{e_{\eta_2}}^{1}(-h) + C_{2V}^{22} N_{e_{\eta_2}}^{1}(-h) \right] \\
+ M_{o_{\eta_2}}(h) \left[ C_{3H}^{22} M_{e_{\eta_2}}^{1}(-h) + C_{3H}^{22} M_{e_{\eta_2}}^{1}(-h) \right] \\
+ N_{o_{\eta_2}}(h) \left[ C_{3V}^{22} N_{e_{\eta_2}}^{1}(-h) + C_{3V}^{22} N_{e_{\eta_2}}^{1}(-h) \right] \right\}.$$
and for $f = 3$

\[
\mathcal{G}_{es}^{(32)}(r, r') = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{(2 - \delta^0_n)}{\eta^2_n} \left\{ M_{o\eta_3}^e(h) \left[ C_{3H}^{32} M_{o\eta_2}^e(-h) + C_{3H}^{32} M_{o\eta_2}^e(-h) \right] \right. \\
+ \left. N_{o\eta_3}^e(h) \left[ C_{3V}^{32} N_{o\eta_2}^e(-h) + C_{3V}^{32} N_{o\eta_2}^e(-h) \right] \right. \\
+ \left. N_{o\eta_3}^o(h) \left[ C_{4H}^{32} M_{o\eta_2}^o(-h) + C_{4H}^{32} M_{o\eta_2}^o(-h) \right] \right. \\
+ \left. M_{o\eta_3}^o(h) \left[ C_{4V}^{32} N_{o\eta_2}^o(-h) + C_{4V}^{32} N_{o\eta_2}^o(-h) \right] \right\}. \quad (30c)
\]

Degenerating the results shown in Eqs. (16) and (17) and following the procedure similar to the above, the coefficients of scattered DGF can be obtained as well. They are given by:

\[
C_{11}(H,V) = -\frac{1}{D_3} \left( T_{12}^{1(H,V)} R_{2(21)}^{(H,V)} - T_{22}^{1(H,V)} R_{2(11)}^{(H,V)} \right), \quad (31a)
\]

\[
C_{12}(H,V) = \frac{1}{D_3} \left( T_{12}^{2(H,V)} R_{2(23)}^{(H,V)} - T_{22}^{2(H,V)} R_{2(13)}^{(H,V)} \right), \quad (31b)
\]

\[
C_{21}(H,V) = -\frac{1}{D_3} \left( T_{21}^{1(H,V)} R_{2(21)}^{(H,V)} - T_{11}^{1(H,V)} R_{2(21)}^{(H,V)} \right), \quad (31c)
\]

\[
C_{22}(H,V) = \frac{1}{D_3} \left( T_{21}^{2(H,V)} R_{2(13)}^{(H,V)} - T_{11}^{2(H,V)} R_{2(13)}^{(H,V)} \right), \quad (31d)
\]

\[
C_{11}(H,V) = R_{1(11)}^{(H,V)} C_{11}(H,V) + R_{1(12)}^{(H,V)} C_{21}(H,V) - R_{1(11)}^{(H,V)}, \quad (31e)
\]

\[
C_{12}(H,V) = R_{1(11)}^{(H,V)} C_{12}(H,V) + R_{1(12)}^{(H,V)} C_{22}(H,V), \quad (31f)
\]

\[
C_{21}(H,V) = R_{1(21)}^{(H,V)} C_{12}(H,V) + R_{1(22)}^{(H,V)} C_{21}(H,V), \quad (31g)
\]

\[
C_{22}(H,V) = R_{1(21)}^{(H,V)} C_{12}(H,V) + R_{1(22)}^{(H,V)} C_{22}(H,V), \quad (31h)
\]

\[
C_{32}(H,V) = R_{1(31)}^{(H,V)} C_{12}(H,V) + R_{1(32)}^{(H,V)} C_{22}(H,V), \quad (31i)
\]
6.3 Current Source in the Last Layer

When the excitation current is located in the last layer of the three-layers, the dyadic Green's function can be given below:

for $f = 1$

\[
\overline{G}^{(13)}_{es}(\mathbf{r}, \mathbf{r}') = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{2 - \delta_{n0}}{\eta_n^2} \left[ C_{1H}^{13} M_{e,\eta_1}^{(1)} (h) M_{e,\eta_3}^{(1)} (-h) + C_{1V}^{13} N_{e,\eta_1}^{(1)} (h) \right. \\
+ \left. C_{2H}^{13} M_{e,\eta_3}^{(1)} (h) N_{e,\eta_3}^{(1)} (-h) + C_{2V}^{13} M_{e,\eta_3}^{(1)} (h) N_{e,\eta_3}^{(1)} (-h) \right],
\]

for $f = 2$

\[
\overline{G}^{(23)}_{es}(\mathbf{r}, \mathbf{r}') = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{2 - \delta_{n0}}{\eta_n^2} \left[ C_{1H}^{23} M_{e,\eta_2}^{(1)} (h) M_{e,\eta_3}^{(1)} (-h) + C_{1V}^{23} N_{e,\eta_2}^{(1)} (h) \right. \\
+ \left. C_{2H}^{23} N_{e,\eta_2}^{(1)} (h) M_{e,\eta_3}^{(1)} (-h) + C_{2V}^{23} M_{e,\eta_2}^{(1)} (h) N_{e,\eta_3}^{(1)} (-h) \right].
\]
and for \( f = 3 \)

\[
\overline{G}^{(33)}_{es}(\mathbf{r}, \mathbf{r'}) = \frac{i}{8\pi} \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} \frac{(2 - \delta_n^0)}{\eta_3^2} \left[ C^{33}_{3H} M_{e,n_0}^{o,n_3}(h) M_{e,n_0}^{o,n_3}(-h) + C^{33}_{3V} N_{e,n_0}^{o,n_3}(h) N_{e,n_0}^{o,n_3}(-h) \right] + C^{33}_{4H} M_{e,n_0}^{o,n_3}(h) N_{e,n_0}^{o,n_3}(-h). \tag{32c}
\]

The coefficients are:

\[
C^{13}_{1(H,V)} = \frac{1}{D_3} T^{1(H,V)}_{22}, \tag{33a}
\]

\[
C^{13}_{2(H,V)} = -\frac{1}{D_3} T^{1(H,V)}_{21}, \tag{33b}
\]

\[
C^{23}_{1(H,V)} = \mathcal{R}^{(H,V)}_{1(11)} c^{13}_{1(H,V)} + \mathcal{R}^{(H,V)}_{1(12)} c^{13}_{2(H,V)}, \tag{33c}
\]

\[
C^{23}_{2(H,V)} = \mathcal{R}^{(H,V)}_{1(21)} c^{13}_{1(H,V)} + \mathcal{R}^{(H,V)}_{1(22)} c^{13}_{2(H,V)}, \tag{33d}
\]

\[
C^{23}_{3(H,V)} = \mathcal{R}^{(H,V)}_{1(31)} c^{13}_{1(H,V)} + \mathcal{R}^{(H,V)}_{1(32)} c^{13}_{2(H,V)}, \tag{33e}
\]

\[
C^{23}_{4(H,V)} = \mathcal{R}^{(H,V)}_{1(41)} c^{13}_{1(H,V)} + \mathcal{R}^{(H,V)}_{1(42)} c^{13}_{2(H,V)}, \tag{33f}
\]

\[
C^{33}_{3(H,V)} = T^{1(H,V)}_{31} C^{13}_{1(H,V)} + T^{1(H,V)}_{32} C^{13}_{2(H,V)} - 1, \tag{33g}
\]

\[
C^{33}_{4(H,V)} = T^{1(H,V)}_{41} C^{13}_{1(H,V)} + T^{1(H,V)}_{42} C^{13}_{2(H,V)}. \tag{33h}
\]

So far, two applications of the general formulation given earlier have been made. The scattering DGFs and their coefficients for all the regions of a two-layered medium and a three-layered medium have been provided for future use. The source is assumed in various regions of the geometries considered. All the modes, rather than the single mode \( n = 0 \) in [1], have been included. Readers, who want to formulate
the electromagnetic fields due to either a known source distribution or a unknown surface distribution, may straightforwardly make use of the DGFs and their scattering coefficients in their analysis without concerning themselves with the boundary match on the dielectric cylindrical interfaces.

7. CONCLUSIONS

As an amendment of the paper with critical mistakes published by Xi-ang and Lu [1], this paper presents the correct form of electromagnetic dyadic Green’s functions in spectral domain for cylindrically multilayered media. Not only correcting the mistakes occurring in [1], this paper aims at also deriving the general scattering coefficients of the scattering DGFs. In the general formulation, the source and observation points are located arbitrarily in a layer of the multilayered cylinder, the layer number of the geometry is also arbitrary. Different from the authors’ previous procedure [11] for obtaining the general coefficients of the scattering DGFs, a unified method is developed regardless of the location of the source point. For specific applications, the DGFs and their coefficients for all the modes are formulated and represented for all the regions of a cylindrically two-layered and three-layered media, instead of those contributed by the single but non-dominant mode (where \( n = 0 \)) and deduced by Xiang and Lu [1]. The amendment can avoid the possible misleading resulted by [1] and assist readers who need the DGFs in cylindrically multilayered geometry to analyze antenna radiation due to a microstrip patch, a dipole, or any other type embedded in a dielectric-coated multilayered cylinder. The general results of the paper can also be used to analyze various wave modes (hybrid direct and multi-hop reflected waves) and their propagation mechanisms (surface wave mixed paths) in optical fibers and different tunnels.

Finally, we summarize the procedure for obtaining the coefficients of the scattering DGFs as follows:

- If the specific geometry and the dielectric characteristics of the medium are known, the parameters \( k_j, \tau_j, \zeta_{jm} \) and \( q_j \) \((j, m = 1, 2, \ldots, N)\) in (11), and the elements \( R_{j(m,n)}^{(H,V)} \) \((m, n = 1, 2, 3, 4)\) shown in (12) can be determined.
• By using the above mentioned intermediate parameters, the elements $T^K_{mn}^{H,V}$ ($K = 1, 2, \ldots, N - 1$) shown in (14) can be calculated.

• After obtaining the above elements, the coefficients of the scattering Green dyadics shown in (8) can be derived by means of matrix recurrence equations. They are given by the matrix equations (16), (17) for the first and last layers and by the recurrent equations (15) for the rest of the layers.

• The electric type of Green dyadic for each layer of the multilayered cylinder with the known layer number is considered as the sum of the unbounded and the scattering DGFs. The scattering DGF can be constructed by making use of the formula in (6).

This procedure may conveniently assist readers to write their own algorithms.

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