MULTIPLE SCATTERING BY AN ARRAY OF FINITE CYLINDRICALLY CURVED THIN SCREENS

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Abstract—The electromagnetic scattering by an array of finite cylindrically curved thin screens is investigated in this paper, and the normally incident plane wave can be either TM₂- or TE₂-polarised. The mathematical treatment here is based on the direct integral equation technique combined with the Galerkin's procedure, and the induced currents on the curved thin screens or apertures are expanded in terms of a series of Chebyshev polynomials of the first kind. Furthermore, the far-zone field solutions to such a multiple coupling system are obtained in an analytical form, and parametric calculations are performed to show the variation of the scattered far-zone field distributions at both TM₂- and TE₂-polarisation incidences.

1. INTRODUCTION

In the past a few decades, the interaction of electromagnetic waves with some special cavity-backed screens or apertures has received considerable attention, and this study has been mainly motivated by developing new techniques for solving penetration and shielding problems in the fields of electromagnetic engineering. Among those who made significant progress, Harrington and Mautz have presented a general procedure for formulating problems involving electromagnetic interaction with single aperture in conducting bodies as well as the electromagnetic penetration into a conducting circular cylinder through a narrow slot using the method of moments [1–4], Ziolkowski and Grant have studied the scattering by cavity-backed cylindrical apertures and an axially slotted infinite cylinder using the generalized dual series method [5, 6], Felsen and Vecchi have developed a resonant mode expansion for wave
scattering from slit coupled cylindrical cavities with interior loading [7], and more recently, Shumpert and Butler have analyzed the penetration of both TM and TE waves through single slots in conducting cylinders [8, 9]. On the other hand, the problem of multiple scattering of an incident plane wave by an array of parallel non-overlapping circular cylinders has also been extensively studied [10–13]. In fact, many complicated practical problems require a number of cylinders to model actual scattering environments, so as to improve as well as control scattering properties of objects [14].

In the present work, we will focus on multiple scattering characteristics of an array of finite cylindrically curved thin screens. To the authors' best knowledge, till now very little work has been done concerning with such a generalized multiple interaction system in spite of the study of Veremey and Mittra recently [10]. Here, the normally incident plane wave can be either $TM_z$- or $TE_z$-polarised. The mathematical treatment is based on the technique of direct integral equation. This hybrid technique has been proven to be very flexible for analyzing the electromagnetic characteristics of many screen- or strip-loaded cylindrical dielectric structures, such as waveguides, antenna radiation as well as electromagnetic scattering.

2. GEOMETRIES OF THE PROBLEM

Fig. 1 shows the cross section of a planar array made of $N$ parallel, infinitely long, perfectly conducting, cylindrically curved screens in free space, and the radii and angular ranges of these screens are denoted by $R_p$ and $[\psi_1^{(p)}, \psi_2^{(p)}]$, respectively. The separation between screens $p$ and $q$ ($q \neq p$, $q = 1, 2, \ldots, p - 1, p + 1, \ldots, N$) is determined by $D_{qp}$, and practically $D_{qp} > (R_p + R_q)$.

3. SOLUTION METHODOLOGY

3.1 The Case of $TM_z$-Polarisation

In Fig. 1, the excitation is provided by a plane EM wave of $TM_z$-polarisation at first. With respect to the co-ordinates system $O_p(\rho_p, \varphi_p, z)$ of the $p$th screen, the normally incident wave is expressed as ($e^{i\omega t}$):
Figure 1. Geometry and coordinates of a random array of $N$ cylindrically curved thin screens.

$$E_{z}^{(inc)}(\rho_{p}, \varphi_{p}) = E_{0}e^{ik_{0}\rho_{p}'\cos(\varphi_{p}' - \varphi_{0})} \sum_{n=-\infty}^{+\infty} i^{n} J_{n}(k_{0}\rho_{p})e^{in(\varphi_{p} - \varphi_{0})},$$

$$p = 1, 2, \ldots, N \quad (1)$$

where $k_{0} = \omega\sqrt{\varepsilon_{0}\mu_{0}}$ is the wavenumber in free space, and $J_{n}(k_{0}\rho_{p})$ is the $n$th-order Bessel function of the first kind.

In order to understand the scattered field characteristics of the array in Fig. 1, one needs to evaluate the following well-known integrals consisting of the induced electric currents on all screens:

$$E_{z}^{(p)}(\rho_{p}, \varphi_{p}) = i\omega\mu_{0} \iiint G_{zz}^{(p)}(\rho_{p}, \varphi_{p} | R_{p})J_{zz}^{(p)}(R_{p})ds',$$

$$p = 1, 2, \ldots, N \quad (2)$$

where $G_{zz}^{(p)}(\rho_{p}, \varphi_{p} | R_{p})$ ($p = 1, 2, \ldots, N$) represents the $zz$-component of electric dyadic Green’s function due to the induced electric current, and it can be written as
where \( \delta_{n0} = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \). Furthermore, the induced electric currents on screens can be expressed as

\[
J_{ex}^{(p)}(R_p) = J_{exp}(\varphi') \delta(\rho_p - R_p),
\]

and the unknown function \( \{ J_{exp}(\varphi') \} \) is expanded conveniently in terms of a series of Chebyshev polynomials [15], i.e.,

\[
J_{exp}(\varphi') = \sum_{l=0}^{+\infty} a_{cl}^{(p)} \frac{T_l(x'_p)}{\sqrt{1 - x'^2_p}} \left[ U \left( \varphi' - \psi_2^{(p)} \right) - U \left( \varphi' - \psi_1^{(p)} \right) \right],
\]

where \( \{ T_l(\cdot) \} \) represents the Chebyshev polynomials of the first kind, \( U(x' - x) = \begin{cases} 1, & x' \geq x \\ 0, & x' < x \end{cases} \), and

\[
x'_p = \left[ \varphi' - \frac{\psi_2^{(p)} + \psi_1^{(p)}}{2} \right] / \frac{\psi_2^{(p)} - \psi_1^{(p)}}{2}.
\]

In (4b), \( \{ a_{cl}^{(p)} \} \) stands for the unknown expanding coefficients to be determined, and the factor, \( \frac{1}{\sqrt{1 - x'^2_p}} \), accounts for the edge effects of each screen. On the \( p \)th screen at \( \rho_p = R_p \), we have the boundary condition as follows:

\[
E_z^{(p)}(\rho_p, \varphi_p) + \sum_{q=1}^{N} E_z^{(q)}(\rho_p, \varphi_p) + E_z^{(inc)}(\rho_p, \varphi_p) = 0,
\]

(6a)
where

\[
E_z^{(p)}(\rho_p, \varphi_p) = -\frac{\pi \omega \mu_0 R_p}{4} \sum_{n=0}^{+\infty} \sum_{l=0}^{+\infty} (2 - \delta_{n0}) (-1)^l J_n(k_0 R_p) H_n^{(2)}(k_0 \rho_p) \cdot \Phi_{21-}^{(p)} \left[ \cos(n \varphi_p) a_e(n, l, p) + \sin(n \varphi_p) b_e(n, l, p) \right] ,
\]

\[
a_e(n, l, p) = a_{e2l}^{(p)} \cos \left( n \Phi_{21+}^{(p)} \right) J_{2l} \left( n \Phi_{21-}^{(p)} \right) - a_{e2l+1}^{(p)} \sin \left( n \Phi_{21+}^{(p)} \right) J_{2l+1} \left( n \Phi_{21-}^{(p)} \right) ,
\]

\[
b_e(n, l, p) = a_{e2l}^{(p)} \sin \left( n \Phi_{21+}^{(p)} \right) J_{2l} \left( n \Phi_{21-}^{(p)} \right) + a_{e2l+1}^{(p)} \cos \left( n \Phi_{21+}^{(p)} \right) J_{2l+1} \left( n \Phi_{21-}^{(p)} \right) ,
\]

while \( \Phi_{21\pm}^{(p)} = \frac{\psi_{2\pm}^{(p)} \pm \psi_{1\pm}^{(p)}}{2} \), and \( J_{2l}(\cdot) \) and \( J_{2l+1}(\cdot) \) are Bessel functions of the first kind. In (6a), \( E_z^{(q)}(\rho_p, \varphi_p) \) is the electric field \( z \)-component radiated by the \( q \)th screen with respect to the co-ordinates system \( O_p(\rho_p, \varphi_p, z) \), and the translational addition theorem for Hankel function of the second kind must be exploited here, i.e.,

\[
H_n^{(2)}(k_0 \rho_q) \cos(n \varphi_q) = \sum_{m=0}^{+\infty} J_m(k_0 \rho_p) \left[ T_{1c}^{(pq)} \cos(m \varphi_p) + T_{1s}^{(pq)} \sin(m \varphi_p) \right] ,
\]

\[
H_n^{(2)}(k_0 \rho_q) \sin(n \varphi_q) = \sum_{m=0}^{+\infty} J_m(k_0 \rho_p) \left[ T_{2c}^{(pq)} \cos(m \varphi_p) + T_{2s}^{(pq)} \sin(m \varphi_p) \right] ,
\]

where

\[
T_{1c}^{(pq)} = H_m^{(2)}(k_0 D_{pq}) \cos \left( (m - n) \varphi_p \right) + (1 - \delta_{m0}) H_{m+n}^{(2)}(k_0 D_{pq}) \cos \left( (m + n) \varphi_p \right) ,
\]

\[
T_{1s}^{(pq)} = H_m^{(2)}(k_0 D_{pq}) \sin \left( (m - n) \varphi_p \right) + (1 - \delta_{m0}) H_{m+n}^{(2)}(k_0 D_{pq}) \sin \left( (m + n) \varphi_p \right) ,
\]

\[
T_{2c}^{(pq)} = -H_{m-n}^{(2)}(k_0 D_{pq}) \sin \left( (m - n) \varphi_p \right) + (1 - \delta_{m0}) H_{m+n}^{(2)}(k_0 D_{pq}) \sin \left( (m + n) \varphi_p \right) ,
\]

\[
T_{2s}^{(pq)} = H_{m-n}^{(2)}(k_0 D_{pq}) \cos \left( (m - n) \varphi_p \right) - (1 - \delta_{m0}) H_{m+n}^{(2)}(k_0 D_{pq}) \cos \left( (m + n) \varphi_p \right) .
\]
\[ \delta_{m0} = \begin{cases} 1 & m = 0, \\ 0 & m \neq 0. \end{cases} \quad D_{pq}^2 = \rho_p' \cos^2 \varphi_p - \rho_q' \cos \varphi_q, \]

and

\[ \varphi_{pq}' = \begin{cases} \cos^{-1} \left( \frac{\rho_p' \cos \varphi_p - \rho_q' \cos \varphi_q}{D_{pq}} \right), & \rho_p' \sin \varphi_p \geq \rho_q' \sin \varphi_q, \\ -\cos^{-1} \left( \frac{\rho_p' \cos \varphi_p - \rho_q' \cos \varphi_q}{D_{pq}} \right), & \rho_p' \sin \varphi_p < \rho_q' \sin \varphi_q. \end{cases} \]

Therefore, in the co-ordinates system \( O_p(\rho_p, \varphi_p, z) \),

\[ E^{(q)}_z(\rho_p, \varphi_p) = -\frac{\pi \omega \mu_0 R_q}{4} \sum_{n=0}^{+\infty} \sum_{l=0}^{+\infty} \sum_{m=0}^{+\infty} (2 - \delta_{n0}) \]

\[ \cdot (-1)^l J_n(k_0 R_q) J_m(k_0 \rho_p) \Phi_{21-}^{(q)} \]

\[ \cdot \left\{ \left[ T_{1c}^{(pq)} \cos(m \varphi_p) + T_{1s}^{(pq)} \sin(m \varphi_p) \right] a_e(n, l, q) \right. \]

\[ + \left. \left[ T_{2c}^{(pq)} \cos(m \varphi_p) + T_{2s}^{(pq)} \sin(m \varphi_p) \right] b_e(n, l, q) \right\}; \quad (8) \]

Furthermore, by substituting (1a), (6b) and (8) into (6a) and following the Galerkin’s procedure, i.e., both sides of (6a) are multiplied by the factor \( \frac{T_r(X_r)}{\sqrt{1 - X_r^2}} \) for the \( p \)-th screen and integration over the width of this screen \( X_r = \left( \varphi - \Phi_{21+}^{(r)} \right) / \Phi_{21-}^{(r)} \), it results in a linear algebraic system of infinite order satisfied by \( \{ a_{e2l}, a_{e2l+1} \} \), i.e.,

\[ \sum_{n=0}^{+\infty} \sum_{l=0}^{+\infty} (2 - \delta_{n0})(-1)^l R_p J_n(k_0 R_p) H_n^{(2)}(k_0 R_p) \Phi_{21-}^{(pq)} \tilde{a}_e(n, r, p) a_e(n, l, p) \]

\[ + \sum_{q=1}^{N} \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} \sum_{l=0}^{+\infty} (2 - \delta_{n0})(-1)^l R_q J_n(k_0 R_q) J_m(k_0 R_p) \Phi_{21-}^{(pq)} \tilde{a}_e(m, r, p) \]

\[ \cdot \left[ T_{1c}^{(pq)} a_e(n, l, q) + T_{2c}^{(pq)} b_e(n, l, q) \right] \]

\[ = C e^{ik_0 \rho_p' \cos(\varphi_p - \varphi_0)} \sum_{n=0}^{+\infty} i^n (2 - \delta_{n0}) J_n(k_0 R_p) \tilde{a}_e(n, r, p) \cos(n \varphi_0), \quad (9a) \]
Following a procedure similar to that used above, it can be found that for the qth screen, the unknown expanding coefficients $I$ satisfy the following equations:

\[
\sum_{n=0}^{\infty} \sum_{l=0}^{\infty} (2 - \delta_{n0})(-1)^l R_p J_n(k_0 R_p) H_n^{(2)}(k_0 R_p) \Phi_{21-}^{(p)} \bar{b}_e(n, r, p) b_e(n, l, p) \\
+ \sum_{q=1}^{N} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} (2 - \delta_{n0})(-1)^l R_q J_n(k_0 R_q) J_m(k_0 R_p) \Phi_{21-}^{(q)} \bar{b}_e(m, r, p) \\
\cdot \left[ T_{1s}^{(pq)} a_e(n, l, q) + T_{2s}^{(pq)} b_e(n, l, q) \right]
\]

\[
= C e^{i k_0 \rho'_p \cos(\varphi'_p - \varphi_0)} \sum_{n=0}^{\infty} i^n (2 - \delta_{n0}) J_n(k_0 R_p) \bar{b}_e(n, r, p) \sin(n \varphi_0); \quad (9b)
\]

where $C = \frac{4E_0}{\pi \omega \mu_0}$,

\[
\tilde{a}_e(m, r, p) = \cos \left( m \Phi_{21+}^{(p)} \right) J_{2r} \left( m \Phi_{21+}^{(p)} \right) - \sin \left( m \Phi_{21+}^{(p)} \right) J_{2r+1} \left( m \Phi_{21+}^{(p)} \right), \quad (9c)
\]

\[
\tilde{b}_e(m, r, p) = \sin \left( m \Phi_{21+}^{(p)} \right) J_{2r} \left( m \Phi_{21+}^{(p)} \right) + \cos \left( m \Phi_{21+}^{(p)} \right) J_{2r+1} \left( m \Phi_{21+}^{(p)} \right). \quad (9d)
\]

Following a procedure similar to that used above, it can be found that for the qth screen, the unknown expanding coefficients $\{a_e^{(q)}, a_e^{(q+1)}\}$, satisfy the following equations:

\[
\sum_{n=0}^{\infty} \sum_{l=0}^{\infty} (2 - \delta_{n0})(-1)^l R_q J_n(k_0 R_q) H_n^{(2)}(k_0 R_q) \Phi_{21-}^{(q)} \tilde{a}_e(n, r, q) a_e(n, l, q) \\
+ \sum_{q=1}^{N} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} (2 - \delta_{n0})(-1)^l R_s J_n(k_0 R_s) J_m(k_0 R_q) \Phi_{21-}^{(s)} \tilde{a}_e(m, r, q) \\
\cdot \left[ T_{1c}^{(qs)} a_e(n, l, s) + T_{2c}^{(qs)} b_e(n, l, s) \right]
\]

\[
= C e^{i k_0 \rho'_q \cos(\varphi'_q - \varphi_0)} \sum_{n=0}^{\infty} i^n (2 - \delta_{n0}) J_n(k_0 R_q) \tilde{a}_e(n, r, q) \cos(n \varphi_0); \quad (10a)
\]

\[
\sum_{n=0}^{\infty} \sum_{l=0}^{\infty} (2 - \delta_{n0})(-1)^l R_q J_n(k_0 R_q) H_n^{(2)}(k_0 R_q) \Phi_{21-}^{(q)} \tilde{b}_e(n, r, q) b_e(n, l, q)
\]
By solving (9a,b) and (10a,b) \((s = 1, \ldots, s = q, \ldots, N)\), all the unknown expanding coefficients for determining the induced electric current distributions of screens can be obtained numerically. There are a few issues that are emphasised below.

1. In (9a,b) and (10a,b), not only the edge effect of each screen but also the coupling effect among multiple screens are both taken into account.

2. The above linear equations are solved for each \(q\) independently and all elements are evaluated analytically. For an \(N\)-screen array, there exist \(N\)-sets of unknown current expanding coefficients \(\{a_{e2l}^{(N)}, a_{e2l+1}^{(N)}\}\) to be determined. In order to find a numerical solution of \(\{a_{e2l}^{(N)}, a_{e2l+1}^{(N)}\}\), all the system equations (9a,b) and (10a,b) must be truncated to a finite size.

3. Physical intuition suggests that the truncation numbers \(\{n(r) = 0, \ldots, N_0\}\) depend mainly on the electric dimension \(\max \{k_0 R N\}\). Usually, \(N_0\) becomes not very large for the electrically non-large screens, so that the numerical solution is feasible.

4. For different angular widths of multiple screens, the multiple truncation numbers \(\{l = 0, \ldots, L_{1(e)}^{(e)}, \ldots, N\}\) can be different, and the integer numbers \(\{L_{1(e)}^{(e)}\}\) required for all screens are directly related to their angular widths.

As soon as the electric currents on all screens are calculated from (9a,b)–(10a,b), the far-zone field excited by screen currents can be calculated through

\[
E_2^{(\text{total})} \approx \sqrt{\frac{2i}{\pi k_0 \rho}} e^{ik_0 \rho} F_\epsilon(\varphi), \quad \text{(11a)}
\]
where

\[ F_\text{c}(\varphi) = -\frac{\pi \omega \mu_0}{4} \sum_{p=1}^{N} \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} (2 - \delta_{n0})(-1)^l e^{ik_0 \rho_p' \cos(\varphi'_p - \varphi)} J_n(k_0 R_p) \]
\[ \cdot \left[ \cos(n \varphi) a_\text{e}(n, l, p) + \sin(n \varphi) b_\text{e}(n, l, p) \right]. \] (11b)

### 3.2 The Case of TEz-Polarisation

In Fig. 1, when the incident plane wave is of a TEz-Polarisation, the incident magnetic field component with respect to the co-ordinates system can be written as

\[ H^{(\text{inc})}_{z p}(\rho_p, \varphi_p) = H_0 e^{ik_0 \rho_p' \cos(\varphi'_p - \varphi_0)} \sum_{n=-\infty}^{+\infty} i^n J_n(k_0 \rho_p) e^{in \varphi - \varphi_0}, \]
\[ p = 1, 2, \ldots, N. \] (13)

The induced magnetic fields excited by magnetic currents on all cylindrical apertures \([\tilde{\psi}_1(p), \tilde{\psi}_2(p)](\tilde{\psi}_1(p) = \psi_2(p), \tilde{\psi}_2(p) = \psi_1(p))\) is calculated by

\[ H_z^{(p)}(\rho_p, \varphi_p) = i \omega \varepsilon_0 \int \mathcal{G}_{mzz}(\rho_p, R_p) J_{mzp}(R_p) ds', \] (14a)

and

\[ J_{mzp}(R_p) = J_{mzp}(\varphi') \delta(\rho_p - R_p), \quad \tilde{\psi}_1(p) \leq \varphi' \leq \tilde{\psi}_2(p). \] (14b)

The \(zz\)-component of the magnetic dyadic Green's function \(G_{mzz}^{(p)}(\rho_p | R_p)\) takes the same form as that in (3). On the \(p\)th aperture, the unknown magnetic current \(J_{mzp}(\varphi')\) is also expanded in terms of a series of Chebyshev polynomials as

\[ J_{mzp}(\varphi') = \sum_{l=0}^{+\infty} a_{m,l}^{(p)} \frac{T_l(\tilde{x}'_p)}{\sqrt{1 - \tilde{x}'_p^2}} \left[ U(\varphi' - \tilde{\psi}_2(p)) - U(\varphi' - \tilde{\psi}_1(p)) \right], \] (15a)

where

\[ \tilde{x}'_p = \left[ \varphi' - \frac{\tilde{\psi}_2(p) + \tilde{\psi}_1(p)}{2} \right] / \frac{\left| \tilde{\psi}_2(p) - \tilde{\psi}_1(p) \right|}{2}. \] (15b)
On the $p$th aperture at $\rho_p = R_p$, the boundary condition becomes:

$$H_z^{(p)}(\rho_p, \varphi_p) + \sum_{q=1, q\neq p}^{N} H_z^{(q)}(\rho_p, \varphi_p) + H_z^{(nc)}(\rho_p, \varphi_p) = 0. \quad (16)$$

By virtue of duality theorem, i.e., $\Phi^{(p)}_{21} \rightarrow \Phi^{(p)}_{21\pm}$, $\mu_0 \rightarrow \varepsilon_0$, and $E \rightarrow H$ in (6)–(11), and after mathematical treatment similar to the above, not only the aperture magnetic current distributions but also the co-polarised far-zone scattered magnetic field of the multiple cylindrical curved apertures can be quantified.

4. NUMERICAL RESULTS

According to the above mathematical formulations, some computer codes have been developed for calculating the far-zone scattered field components. Obviously, there exist a large number of interesting cases that we can investigate. However, only some typical finite arrays of cylindrical curved thin screens are examined in the following numerical examples. Certainly, the validity and correctness of our codes, including the accuracy and convergence rate, are checked in detail.

At first, Fig. 2 shows the far-zone scattered electric field distributions due to the presence of two adjacent cylindrical screens corresponding to different incident wave directions of TM$_z$-polarisation, respectively.

In Figs. 2(a) and (b), the angular widths of two screens are assumed to be $\Delta \psi_{21}^{(1)} = \Delta \psi_{21}^{(2)} = 270^\circ$, respectively. For both incidences of TM$_z$-polarisation, convergence experiments are made and three sets of truncation numbers are adopted here, i.e., six-, seven- and eight-orders of Chebyshev polynomials are used to expand the induced currents on both screens ($q = 0, 1, \ldots, 6, 7$ and $8$), respectively. It is clear that high accuracy and fast convergence have been achieved in calculating the scattered far-zone field distribution by the Galerkin’s procedure, and $|E_z|$ always reaches the maximum in the forward direction. Since the angular widths of two screens are very large, so it is physically true that the scattered far-zone field distributions above are similar to that of two identical perfectly conducting cylinders shown in [14].

Fig. 3 depicts the far-zone scattered field distributions due to three semi-circular thin screens in the case of both $\varphi_0 = 0^\circ$ and TM$_z$- and TE$_z$-polarisations, respectively.
Figure 2. The far-zone scattered field distributions from two adjacent cylindrically curved thin screens in the case of $E_0 = 1$, $k_0 R_{1,2} = 2\pi$, $k_0 \rho_{1,2}' = 4\pi$, $\varphi_1' = 0.0$, $\varphi_2' = \pi$, and $\psi_1^{(1)} = -135^\circ$, $\psi_2^{(1)} = 135^\circ$, $\psi_1^{(1)} = 45^\circ$, $\psi_2^{(1)} = 315^\circ$, $\{L_{1,2}^{(e)}, N_0\} = \{6, 6, 40: \circ\}; \{7, 7, 40: \circ\}$; and $\{8, 8, 40: \bullet\}$. 
Figure 3. The far-zone scattered field distribution due to three semi-
circular curved thin screens in the case of $E(H)_0 = 1$, $k_0 R_{1,2,3} = \pi$, $k_0 \rho'_1 = 0.0$, $k_0 \rho'_2 = 2\pi$, $k_0 \rho'_3 = 3\pi$, $\varphi'_1 = 0.0$, $\varphi'_2 = \varphi'_3 = \pi$, $\psi_{1,2,3}^{(1)} = -90^\circ$, $\psi_{1,2,3}^{(2)} = 90^\circ$, and $\{ L_{1,2,3}^{(e,h)}; N_0 \} = \{ 4, 4, 4, 40 : \circ \}; \{ 5, 5, 5, 40 : \circ \}; \{ 6, 6, 6, 40 : \bullet \}$. 

In Fig. 3, the angular widths of three screens or three apertures are assumed to be $\Delta \psi_{21}^{(1)} = \Delta \psi_{21}^{(2)} = \Delta \psi_{21}^{(3)} = 180^\circ$, respectively. For either the TM$_z$- or TE$_z$-polarisation, convergence experiments are also made and three sets of truncation numbers are adopted, i.e., four-, five- and six-orders of Chebyshev polynomials are used to expand the induced currents on three screens for TM$_z$- polarisation and three apertures for TE$_z$-polarisation ($q = 0, 1, \cdots, 4, 5, 6$), respectively. For both cases, an excellent convergence has been achieved. It is also found that both $|E_z|$ and $|H_z|$ reach the maxima in the forward direction $\varphi = 180^\circ$.

Finally, Fig. 4 depicts the far-zone scattered field distributions due to five cylindrically curved thin screens corresponding to different incident directions of TM$_z$- and TE$_z$-polarisations, respectively.

In Fig. 4, the angular widths of five cylindrically screens are assumed to be $\Delta \psi_{21}^{(1)} = \Delta \psi_{21}^{(2)} = \Delta \psi_{21}^{(3)} = \Delta \psi_{21}^{(4)} = \Delta \psi_{21}^{(5)} = 300^\circ$ for the incidence of TM$_z$-polarisation with $\varphi_0 = 0^\circ$ and $30^\circ$. Under such circumstances, eight orders of Chebyshev polynomials of the first kind are used and found accurate enough for expanding the induced elec-
Figure 4. The far-zone scattered field distributions due to five cylindrically curved thin screens in the case of $E_0 = 1$, $k_0 R_{1,2,3,4,5} = \pi$, $k_0 \rho_1 = 0.0$, $k_0 \rho_{2,3,4,5} = 3\pi$, $\varphi_1 = \varphi_2 = 0.0$, $\varphi_3 = \pi/2$, $\varphi_4 = \pi$, $\varphi_5 = 3\pi/2$, $\psi_{1,2,3,4,5}^{(1)} = -150^\circ$, $\psi_{1,2,3,4,5}^{(2)} = 150^\circ$, $\{L_{1,2,3,4,5}^{(e)}, N_0\} = \{6, 6, 6, 6, 6, 40: o\}$; $\{7, 7, 7, 7, 7, 40: o\}$; and $\{8, 8, 8, 8, 8, 40: *\}$; and $\{L_{1,2,3,4,5}^{(h)}, N_0\} = \{3, 3, 3, 3, 3, 40: \bullet \text{ and } o\}$.

In this work, the electromagnetic scattering characteristics of an array of finite cylindrically curved thin screens are investigated using the technique of direct integral equation combined with the Calerkin's procedure. In our numerical calculations, Chebyshev polynomials of
the first kind and of suitable order are adopted for expanding the induced currents on multiple cylindrically curved screens or apertures. It is shown that high accuracy and fast convergence rate have been achieved for predicting the far-zone field distributions of various finite array patterns, and furthermore, the above array of cylindrical curved thin screens can be exploited to model complex scattering environment in EMI as well as EMC fields.

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